are unlikely to be successful. The parameter t/y_C must be included in the correlation equation.

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THERMAL CONDUCTIVITY OF PACKED BEDS AND POWDER BEDS

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NOMENCLATURE

- A_{con}, area of continuous phase perpendicular to the xy plane;
- $\bar{a}$, accommodation coefficient;
- B, constant in equation (1);
- C, constant in equation (1);
- D, constant in equation (16);
- E, Young's modulus;
- H, gas pressure in the pore;
- H_0, normal gas pressure, ($9\cdot93 \times 10^4$ N/m^2);
- h, thickness and width of the skeleton of Fig. 1(d);
- h_r, height of particle microroughnesses;
- k, ratio of gas heat capacities at constant pressure and volume;
- k_k, coefficient of particle adhesion;
- k_m, k_n, empirical coefficients, $k_m = (h_r/L) \times 10^3$;
- L, characteristic size of an elementary cell (particle diameter);
- l, characteristic size of a pore;
- n, number of particles in a layer defined in equation (15);
- P_d, discontinuous phase volume fraction;
- Pr, Prandtl number;
- p, pressure of two particles against each other;
- q, integer defined in equation (20);
- R, thermal resistance;
- R', thermal resistance in the microgap;
- R_L, thermal resistance determined by narrowing of the lines of the heat flow in the region adjacent to the place of contact;
- R_0, thermal resistance of the oxidic film;
- R_{sp}, thermal resistance of microroughness at the place of contact;
- r_{sp}, radius of a contact spot;
- T, mean absolute temperature of packed beds or powder beds.

Greek letters

- ΔR, thermal resistance in a layer;
- Δx, small increment in the x axis;
- δ, one-half of h;
- η, constant defined in equation (7);
- Λ_0, gas molecular free path at normal pressure;
- λ, thermal conductivity;
- λ', thermal conductivity in the microgap;
- λ_0, thermal conductivity of gas at normal pressure;

μ, Poisson's ratio;
ν_g, ratio of λ_g/λ_s;
ν'_g, ratio of λ'_g/λ_s.

Subscripts
c, contact;
e, equivalent;
ec, equivalent continuous phase;
g, gas;
gm, gas conduction;
gr, gas radiation;
s, solid.

INTRODUCTION

THE TECHNIQUE reported by the writers for predicting the thermal conductivity of heterogeneous solid mixtures [1] is modified to account for contact resistance. The contact resistance model discussed in reference [2] is combined with the technique. Results of applying the modified technique to powder and packed beds are compared with experimental data. Other studies [3–18] have been considered in preparing this communication.

ANALYSIS

A schematic of a packed bed or powder bed of unit volume is shown in Fig. 1(a). Based on the development in [1], the discontinuous phase, mainly gases in the pores of the packed beds or powder beds, can be rearranged in the continuous phase, mainly particles, and expressed by a parabolic distribution;

$$y = B + Cx^2 \qquad (1)$$

where $B = \sqrt{(3P_d/2)}$ and $C = -4\sqrt{(\frac{2}{3}P_d)}$, as shown in Fig. 1(b).

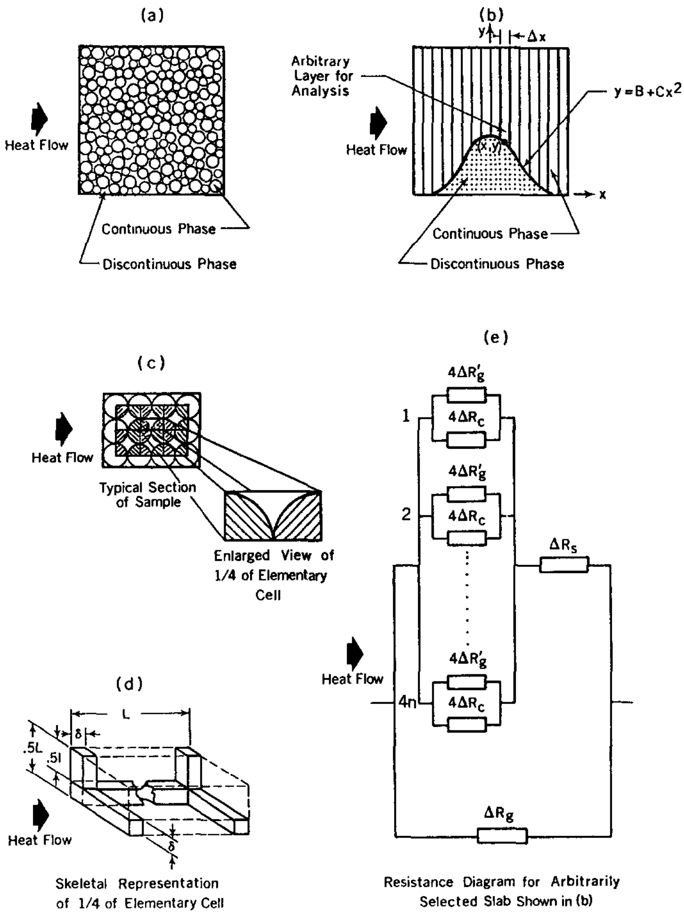

FIG. 1. Model for the study of the thermal conductivity of packed beds and powder beds.

The contact resistance of the particles in the continuous phase can be calculated as follows [2]. Assume the granular particles to be spheres in a cubic arrangement. The elementary cells are shown in Fig. 1(c). Assume the particles to be symmetric. Then, one quarter of an elementary cell, as seen in Fig. 1(c), can be represented in skeletal form as shown in Fig. 1(d). Considering Fig. 1(d), it can be seen that there exists a functional relationship between P_d, the discontinuous phase volume fraction, and h/l, the ratio of the thickness or width of the skeleton to the characteristic size of the pore. This relationship is

$$P_d = f(h/l); \tag{2}$$

where $h = 2\delta$, the thickness and width of the skeleton in Fig. 1(d) and $l = L - h$. The functional relationship plotted of the heat flow in the region adjacent to the place of contact, can be expressed as

$$R_L \doteq \frac{1}{2r_{sp}\lambda_s}; \tag{6}$$

where

$$r_{sp} = 0{\cdot}725\,\sqrt[3]{(\eta p L/2)} \tag{7}$$

and

$$\eta = \frac{2(1-\mu^2)}{E}.$$

Equation (7) is Hertz formula [19]. R_{sp}, which is the resist-

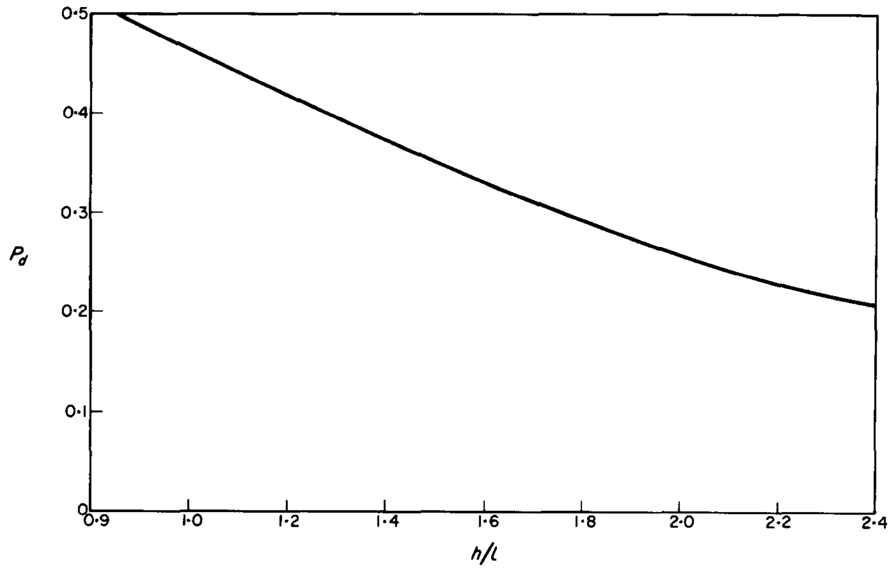

FIG. 2. Graphical representation of P_d as a function of h/l.

resistance in an elementary cell. The contact resistance and the resistance of the gas in the microgap for one quarter of the elementary cell can be designated as R_c and R'_g, respectively. R_c and R'_g can be expressed as

$$R_c = R_L + R_{sp} + R_0 \tag{3}$$

and

$$R'_g = \frac{4h_r k_k}{\lambda'_g h^2} \tag{4}$$

R_c, the contact thermal resistance, is defined as

$$R_c = \frac{1}{\lambda_c L} \tag{5}$$

R_L, R_{sp}, and R_0 in equation (3) will be discussed as follows. R_L, which is the resistance due to the narrowing of the lines ance expressed as

$$R_{sp} = \frac{h_r k_k}{\pi r_{sp}^2 \lambda_s}. \tag{8}$$

h_r in equation (8) is the height of particle microroughnesses. The ratio

$$\frac{h_r}{L} = k_m \cdot 10^{-3} \tag{9}$$

is fairly stable for various sizes of particles. k_m is an empirical coefficient. Equation (8) can be rewritten as

$$R_{sp} = \frac{L k_m k_k}{10^3 \pi r_{sp}^2 \lambda_s}; \tag{10}$$

where k_k is the coefficient of particle adhesion. R_0, which is the resistance of the oxidic film, can be neglected in most cases [20].

λ_g, the thermal conductivity of the gas in a pore, consists of two components, λ_{gr} and λ_{gm}, or

$$\lambda_g = \lambda_{gr} + \lambda_{gm}. \tag{11}$$

Similarly, λ'_g, the thermal conductivity of gas in the microgap, consists of λ'_{gr} and λ'_{gm}, or

$$\lambda'_g = \lambda'_{gr} + \lambda'_{gm}. \tag{12}$$

λ_{gr} and λ'_{gr}, the radiant thermal conductivities of the gas in the pore and in the micrograp, can be calculated from the technique proposed by Loeb [8] or Chudnovsky [21]. For low temperature, λ_{gr} and λ'_{gr} can be neglected. According to Prasolov [22], λ_{gm} can be expressed as

$$\lambda_{gm} = \frac{\lambda_0}{1 + B'/Hl}; \tag{13}$$

where

$$B' = \frac{4k}{k+1}\frac{2-\bar{a}}{\bar{a}}Pr^{-1}\Lambda_0 H_0.$$

Similarly

$$\lambda'_{gm} = \frac{\lambda_0}{1 + B'/k_n Hl}. \tag{14}$$

For simplicity, λ_g can be assumed approximately equal to λ'_g in the calculation.

Now one can proceed to develop an expression for the thermal conductivity of packed beds and powder beds considering contact resistance. Consider an arbitrary layer with thickness Δx as shown in Fig. 1(b). The corresponding resistance diagram can be drawn on Fig. 1(e). The number of particles, n, in the layer can be calculated as

$$\eta = \frac{A_{con}\,\Delta x}{\frac{4}{3}\pi\left(\frac{L}{2}\right)^3}. \tag{15}$$

Table 1. Comparison of predicted thermal conductivities with experimentally determined conductivities for packed beds and powder beds

Mixture	$\lambda_s^\ddagger$	λ_g	P_d	L(mm)	$\lambda_{exp.}$	$\lambda\ddagger_e$	Percentage of deviation	λ§	Percentage of deviation
Steel spheres in air [23]	38·4	0·026	0·38	3·18	0·525 [24]	0·525	0	0·529	0·8
Steel spheres in air [7]	45·0	0·0272	0·413	3·2	0·40	0·51	27·5	0·469	17·3
Steel spheres in air [7]	45·0	0·0272	0·406	3·2	0·60	0·54	−10·0	0·489	−18·5
Plumbum shot in air [7]	34·3	0·0273	0·42	1·6	0·418	0·49	17·2	0·433	3·6
Plumbum shot in air [7]	34·3	0·0273	0·433	6·4	0·404	0·473	17·1	0·39	−3·5
MgO in air at 375°K [21]	24·4	0·0318	0·42	0·268	0·433	0·425	−1·9	0·384	−11·3
MgO in air at 502·4°K [21]	27·9	0·0387	0·42	0·268	0·502	0·515	2·6	0·465	−7·4
MgO in air at 572·1°K [5]	22·1	0·045	0·42	0·268	0·552	0·556	0·7	0·528	−4·4
MgO in air at 723°K [5]	16	0·0533	0·42	0·268	0·661	0·67	1·4	0·602	−8·9
MgO in air at 810°K [5]	13	0·056	0·42	0·268	0·666	0·68	2·1	0·617	−7·4
Average percentage of deviation							8·1		8·3

† Units of λ: W/m deg.
‡ Luikov *et al.* [2].
§ Equation (19).

The total number of quarter elementary cells in the layer is $4n$. ΔR_{ec}, the equivalent resistance of the continuous phase in the layer, can be expressed for the $4n$ quarter elementary cells as

$$\Delta R_{ec} = \Delta R_s + \frac{1}{\underbrace{\left(\frac{1}{4R_c}+\frac{1}{4R'_g}\right)+\left(\frac{1}{4R_c}+\frac{1}{4R'_g}\right)+\ldots+\left(\frac{1}{4R_c}+\frac{1}{4R'}\right)}_{4n \text{ terms}}}$$

$$= \frac{\Delta x}{\lambda_s(1-y)} + \frac{1}{\left(\frac{1}{4R_c}+\frac{1}{4R'_g}\right)\frac{4A_{con}\Delta x}{\frac{4}{3}\pi\left(\frac{L}{2}\right)^3}}$$

$$= \frac{\Delta x}{\lambda_s(1-y)} + \frac{4R_cR'_g}{R'_g+R_c}\cdot\frac{\pi L^3}{24(1-y)\Delta x}$$

$$= \frac{\Delta x}{\lambda_s(1-y)} + \frac{D}{(1-y)\Delta x}; \tag{16}$$

where

$$D = \frac{\pi L^3 R_c R'_g}{6(R'_g + R_c)}.$$

ΔR_e, the equivalent resistance in the layer, can be expressed as

$$\Delta R_e = \frac{\Delta R_g \Delta R_{ec}}{\Delta R_g + \Delta R_{ec}}. \tag{17}$$

Substituting equation (16) into equation (17), yields

$$\Delta R_e = \frac{\frac{\Delta x}{\lambda_g y}\left[\frac{\Delta x}{\lambda_s(1-y)}+\frac{D}{(1-y)\Delta x}\right]}{\left[\frac{\Delta x}{\lambda_s(1-y)}+\frac{D}{(1-y)\Delta x}\right]+\frac{\Delta x}{\lambda_g y}}$$

$$= \frac{\Delta x^3 + D\lambda_s\Delta x}{\lambda_g y\Delta x^2 + D\lambda_g\lambda_s y + \lambda_s(1-y)\Delta x^2} \tag{18}$$

Summing up all x's from $-\frac{1}{2}$ to $\frac{1}{2}$, equation (18) becomes

$$R_e = 2\sum_{0}^{B/2}\frac{\Delta x^3 + D\lambda_s\Delta x}{\lambda_g y\Delta x^2 + D\lambda_g\lambda_s y + \lambda_s(1-y)\Delta x^2} + 2\sum_{B/2}^{1/2}\left(\frac{\Delta x}{\lambda_s}+\frac{D}{\Delta x}\right)$$

$$= 2\sum_{0}^{B/2}\frac{\Delta x^3 + D\lambda_s\Delta x}{\lambda_g y\Delta x^2 + D\lambda_g\lambda_s y + \lambda_s(1-y)\Delta x^2} + \frac{1-B}{\lambda_s} + \frac{4D}{1-B}. \tag{19}$$

The expression for y, the distribution function, can be obtained from equation (1). The value of Δx should be compatible with the value of L. Before computing Δx, q which is some integer should be determined first. q is approximated by

$$q \doteq \frac{B/2}{L}. \tag{20}$$

Once q is determined, Δx can be determined by

$$\Delta x = \frac{B/2}{q}. \tag{21}$$

Mathematically, $q + 1$ represents the number of terms to be summed in equation (19) for the value of Δx determined from equation (21).

Once R_e is determined from equation (19), λ_e is simply the numerical reciprocal of R_e.

A sample problem [2, 23]—steel spheres in air—will be used to illustrate the application of equation (19) in predicting the thermal conductivity of packed beds and powder beds.

Reference data: $L = 3{\cdot}18 \times 10^{-3}$; $P_d = 0{\cdot}38$; $\lambda_s = 38{\cdot}4$; $\lambda_0 = 0{\cdot}026$; $T = 320°\text{K}$; $h/l = 1{\cdot}34$; $h/L = 0{\cdot}573$; $H = 10^5$ N/m^2; $v_g = v'_g = 0{\cdot}677 \times 10^{-3}$; $k_m = 3$; $k_k = 1{\cdot}5$; $\lambda_c = 0{\cdot}03$; experimental equivalent thermal conductivity $\lambda_e = 0{\cdot}525$ W/m deg [24].

Solution: $L = 3{\cdot}18 \times 10^{-3}$ and $h/L = 0{\cdot}573 \Rightarrow h = 0{\cdot}0182$
$v_g = \lambda_g/\lambda_s \Rightarrow \lambda_g = v_g\lambda_s = 0{\cdot}000677 \times 38{\cdot}4 = 0{\cdot}026$.

From equation (5),

$$R_c = \frac{1}{\lambda_c L} = \frac{1}{0{\cdot}03 \times 0{\cdot}00318} = 10480.$$

From equations (4) and (5),

$$R'_g = \frac{4Lk_mk_k}{\lambda'_g h^2 10^3} = \frac{4 \times 0{\cdot}00318 \times 3 \times 1{\cdot}5}{0{\cdot}026 \times (0{\cdot}00182)^2 \times 10^3} = 665.$$

From equation (1), $P_d = 0{\cdot}38 \Rightarrow B = 0{\cdot}755$ and $C = -5{\cdot}3$.

From equation (20), $q \doteq \frac{B/2}{L} \Rightarrow q = 119$.

From equation (21), $\Delta x = \frac{B/2}{q} = 0{\cdot}00317$.

Substituting all the above data into equation (19), yields

$$\lambda_e = 0{\cdot}529 \text{ W/m deg.}$$

which agrees within 0·8 per cent with the experimental value of λ_e. Table 1 compares thermal conductivities of packed beds and powder beds as predicted by equation (19) and thermal conductivity values as obtained by the technique proposed by Luikov *et al.* [2] with experimental data available from the literature. λ_c and λ_g in Table 1 can be obtained experimentally [2] or from equations (5), (11) and (13). The average percentage of deviations of the predicted values from the measured values of thermal conductivity in Table 1 are 8·1 per cent using Luikov *et al.* and 8·3 per cent by equation (19). Deviations in some cases are more than 100 per cent, as shown by Luikov *et al.* [2], when contact resistance is not considered.

SUMMARY

A general equation for predicting the thermal conductivity of packed beds and powder beds has been presented. The equation was developed in terms of the thermal conductivity of the particles and the gas in the pores, discontinuous phase volume ratio, contact resistance, size of the particles, radiative transfer in the gaseous pores, pore size, contact pressure between particles, and surface properties of the particles. Although spherical particles were assumed in the model, the equation can be applied to other particle shapes as long as the mean diameter of the particles can be determined.

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